

三角関数①

【弧度法】

1 次の角を、度数は弧度に、弧度は度数に、それぞれ書き直せ。

- (1) 30° (2) 225° (3) 123° (4) $\frac{4}{3}\pi$ (5) $\frac{\pi}{60}$
- $\frac{1}{6}\pi$ $\frac{5}{4}\pi$ $\frac{41}{60}\pi$ 240° 3°

【扇形の長さや面積】

2 次のような扇形の弧の長さや面積を求めよ。

- (1) 半径4, 中心角 $\frac{\pi}{3}$ $\frac{60}{360} = \frac{1}{6}$ (2) 半径6, 中心角 $\frac{7}{6}\pi$ $\frac{210}{360} = \frac{7}{12}$
- (円) (円)
- 周 $2\pi \cdot 4 = 8\pi$ 周 $2\pi \cdot 6 = 12\pi$
- 面積 $\pi \cdot 4^2 = 16\pi$ 面積 $\pi \cdot 6^2 = 36\pi$
- (扇形) (扇形)
- 弧 $8\pi \times \frac{1}{6} = \frac{4}{3}\pi$ 弧 $12\pi \times \frac{7}{12} = 7\pi$
- 面積 $16\pi \times \frac{1}{6} = \frac{8}{3}\pi$ 面積 $36\pi \times \frac{7}{12} = 21\pi$

【三角関数の値】

3 次の値を、それぞれ求めよ。

- (1) $\cos \frac{5}{4}\pi = -\frac{1}{\sqrt{2}}$ (2) $\sin \frac{11}{6}\pi = -\frac{1}{2}$
- (3) $\tan \frac{4}{3}\pi = -\sqrt{3}$ (4) $\sin(-\frac{\pi}{6}) = -\frac{1}{2}$
- (5) $\cos(-\frac{13}{6}\pi) = \cos(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$
- (6) $\tan(-\frac{9}{4}\pi) = \tan(-\frac{\pi}{4}) = -1$

【三角関数の相互関係】

4 次の値を求めよ。

- (1) θ の動径が第4象限にあり、 $\sin \theta = -\frac{1}{3}$ のとき、 $\cos \theta$ と $\tan \theta$ の値
- $x^2 + 1 = 9$
 $x^2 = 8$
 $x = 2\sqrt{2}$
- $\cos \theta = \frac{2\sqrt{2}}{3}$
 $\tan \theta = -\frac{1}{2\sqrt{2}}$
- (2) θ の動径が第3象限にあり、 $\tan \theta = 3$ のとき、 $\sin \theta$ と $\cos \theta$ の値
- $x^2 = 1 + 9$
 $= 10$
 $x = \sqrt{10}$
- $\sin \theta = -\frac{3}{\sqrt{10}}$
 $\cos \theta = -\frac{1}{\sqrt{10}}$

【相互関係による式の値】

5 $\sin \theta + \cos \theta = \frac{1}{2}$ のとき、次の式の値を求めよ。

- (1) $\sin \theta \cos \theta$
- $2 \sin \theta \cos \theta = \frac{1}{4} - 1$
 $(\sin \theta + \cos \theta)^2 = \frac{1}{4}$
 $\sin^2 + 2 \sin \theta \cos \theta + \cos^2 = \frac{1}{4}$
 $1 + 2 \sin \theta \cos \theta = \frac{1}{4}$
 $2 \sin \theta \cos \theta = -\frac{3}{4}$
- (2) $\sin^3 \theta + \cos^3 \theta$
- $(\sin \theta + \cos \theta)^3 - 3 \sin \theta \cos \theta (\sin \theta + \cos \theta)$
 $= (\frac{1}{2})^3 - 3 \cdot (-\frac{3}{8}) \cdot \frac{1}{2}$
 $= \frac{1}{8} + \frac{9}{16} = \frac{11}{16}$

(3) $\sin \theta - \cos \theta$

$(\sin \theta - \cos \theta)^2 = \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta$

$= 1 - 2 \cdot (-\frac{3}{8})$
 $= \frac{7}{4}$

$\sin \theta - \cos \theta = \pm \frac{\sqrt{7}}{2}$

【相互関係による等式の証明】

6 等式 $\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$ を証明せよ。

(左辺) $= \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta$
 $= \sin^2 \theta (\frac{1}{\cos^2 \theta} - 1)$
 $= \sin^2 \theta \tan^2 \theta =$ (右辺)

\therefore (左辺) = (右辺)

【三角関数のグラフ[1]】

7 次の関数のグラフをかけ。また、その周期を求めよ。

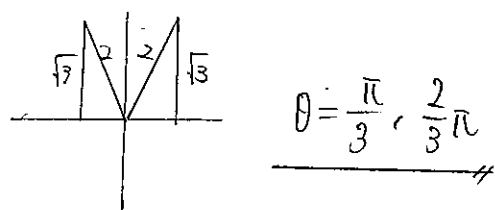
- (1) $y = \frac{1}{2} \sin \theta$ (2) $y = \cos 2\theta$ $\frac{\pi}{2}$
- (3) $y = \sin(\theta + \frac{\pi}{3})$ 2π (4) $y = \cos \theta + 1$ 2π

三角関数②

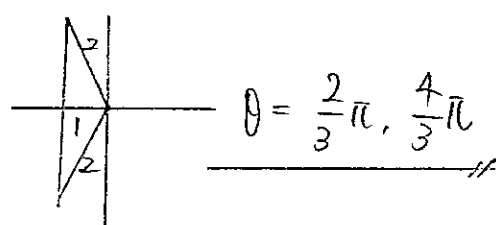
【三角関数を含む方程式】

8 $0 \leq \theta < 2\pi$ のとき、次の方程式を解け。

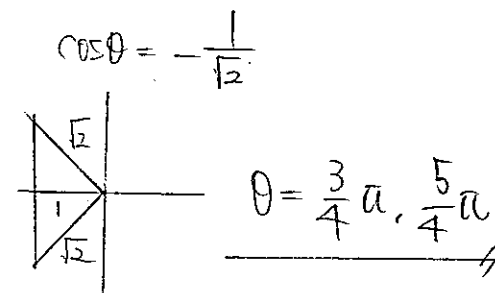
(1) $\sin \theta = \frac{\sqrt{3}}{2}$



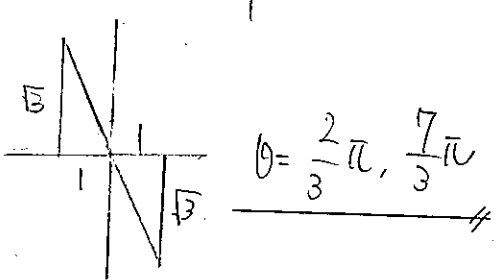
(2) $2\cos \theta + 1 = 0$



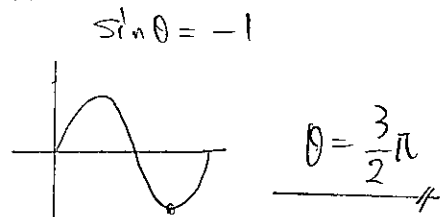
(3) $\cos \theta = -\frac{\sqrt{2}}{2}$



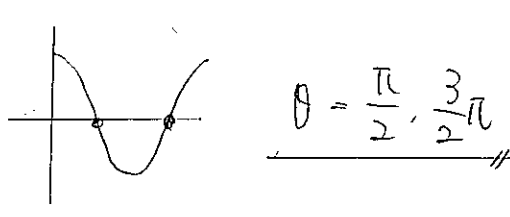
(4) $\tan \theta = -\frac{\sqrt{3}}{1}$



(5) $\sin \theta + 1 = 0$



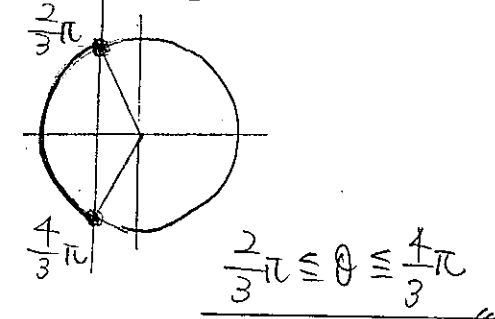
(6) $\cos \theta = 0$



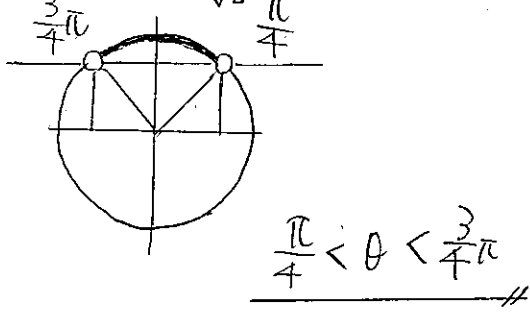
【三角関数を含む不等式】

9 $0 \leq \theta < 2\pi$ のとき、次の不等式を解け。

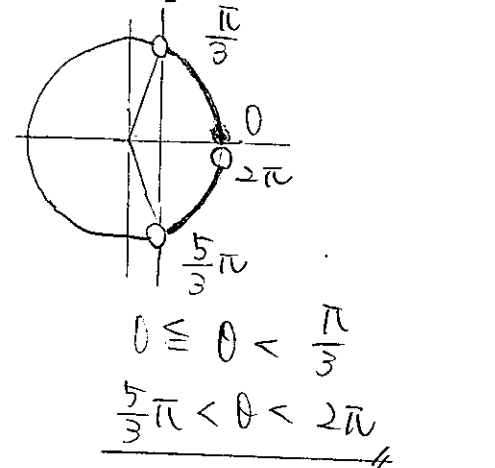
(1) $\cos \theta \leq -\frac{1}{2}$



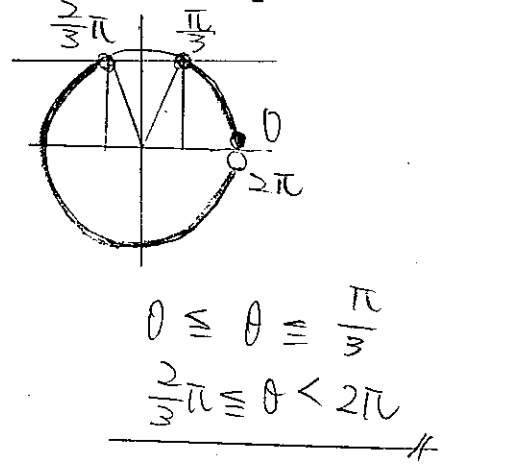
(2) $\sin \theta > \frac{1}{\sqrt{2}}$



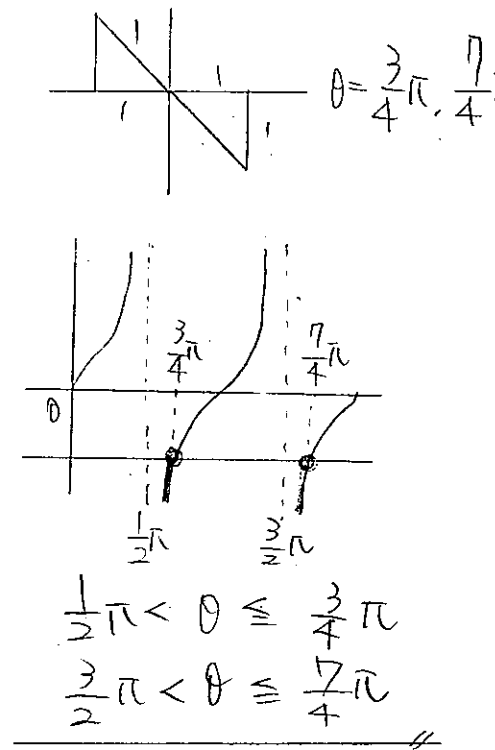
(3) $\cos \theta > \frac{1}{2}$



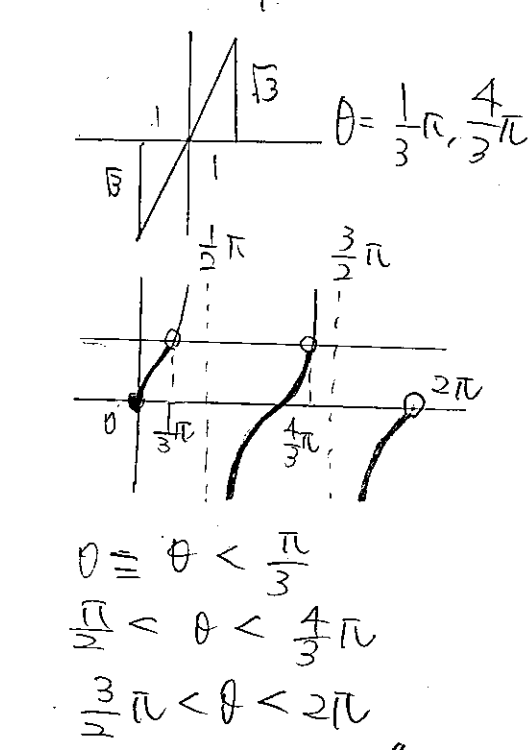
(4) $\sin \theta \leq \frac{\sqrt{3}}{2}$



(5) $\tan \theta \leq -\frac{1}{1}$



(6) $\tan \theta < \frac{\sqrt{3}}{1}$



【三角関数を含む方程式[1]】

10 $0 \leq \theta < 2\pi$ のとき、方程式 $\sin(\theta + \frac{\pi}{3}) = \frac{1}{\sqrt{2}}$ を解け。

$\theta + \frac{\pi}{3} = t$ とおく
 $\sin t = \frac{1}{\sqrt{2}}$
 $0 \leq \theta < 2\pi$
 \downarrow
 $0 + \frac{\pi}{3} \leq \theta + \frac{\pi}{3} < 2\pi + \frac{\pi}{3}$
 \downarrow
 $\frac{\pi}{3} \leq t < \frac{7}{3}\pi$
 $60^\circ \sim 420^\circ$

$t = \frac{\pi}{4}, \frac{3}{4}\pi$
 $45^\circ \quad 135^\circ$
 $\frac{9}{4}\pi$

$t = \frac{3}{4}\pi$ のとき、 $t = \frac{9}{4}\pi$ のとき
 $\theta + \frac{\pi}{3} = \frac{3}{4}\pi$ $\theta + \frac{\pi}{3} = \frac{9}{4}\pi$
 $\theta = \frac{5}{12}\pi$ $\theta = \frac{23}{12}\pi$

【三角関数を含む方程式[2]】

11 $0 \leq \theta < 2\pi$ のとき、方程式 $2\cos^2 \theta + 3\cos \theta - 2 = 0$ を解け。

$\cos^2 \theta = t$ とおく
 $2t^2 + 3t - 2 = 0$
 $(2t - 1)(t + 2) = 0$

$t = \frac{1}{2}, -2$
 不適

$t = \frac{1}{2}$ のとき
 $\cos \theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{2}{3}\pi$

$0 \leq \theta < 2\pi$
 \downarrow
 $-1 \leq \cos \theta \leq 1$
 \downarrow
 $-1 \leq t \leq 1$

【三角関数を含む関数の最大・最小】

12 $0 \leq \theta < 2\pi$ のとき、関数 $y = -\sin^2 \theta - \cos \theta + 1$ の最大値と最小値を求めよ。また、そのときの θ の値を求めよ。

$y = -(1 - \cos^2 \theta) - \cos \theta + 1$
 $y = \cos^2 \theta - \cos \theta$
 $\cos \theta = t$ とおく
 $y = t^2 - t$
 $= (t - \frac{1}{2})^2 - \frac{1}{4}$

$t = z$
 Max 2 ($t = -1$)
 min $-\frac{1}{4}$ ($t = \frac{1}{2}$)

Max $y(-1) = 1 + 1 = 2$ $\cos \theta = -1$
 $\theta = \pi$
 min $-\frac{1}{4}$ $\cos \theta = \frac{1}{2}$
 $\theta = \frac{\pi}{3}, \frac{5}{3}\pi$

従って
 Max 2 ($\theta = \pi$)
 min $-\frac{1}{4}$ ($\theta = \frac{\pi}{3}, \frac{5}{3}\pi$)

三角関数③

【加法定理】

① 加法定理を用いて、次の三角関数の値を求めよ。

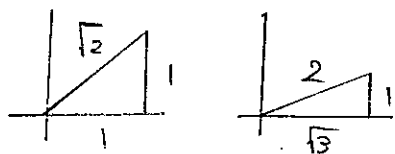
(1) $\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$

$= \frac{\sqrt{6} + \sqrt{2}}{4}$



(2) $\cos 15^\circ = \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$

$= \frac{\sqrt{6} + \sqrt{2}}{4}$

(3) $\tan 105^\circ = \tan(60^\circ + 45^\circ)$

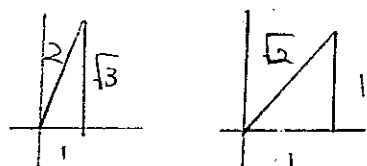
$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ}$

$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$

$= \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$

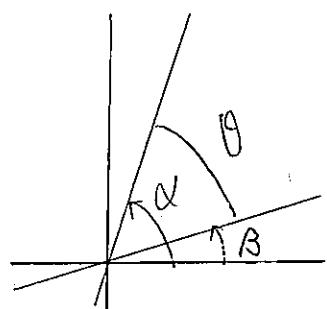
$= \frac{(1 + \sqrt{3})^2}{(1 - \sqrt{3})(1 + \sqrt{3})}$

$= \frac{4 + 2\sqrt{3}}{1 - 3}$



【2直線のなす角】

② 2直線 $y=2x-1$, $y=\frac{1}{3}x+1$ のなす角 θ を求めよ。ただし、 $0 < \theta < \frac{\pi}{2}$ とする。



$\tan \alpha = 2$

$\tan \beta = \frac{1}{3}$

$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

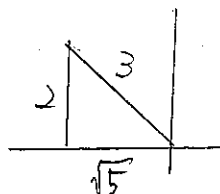
$= \frac{2 - \frac{1}{3}}{1 + \frac{2}{3}}$

$\therefore \theta = \frac{\pi}{4}$

【加法定理の応用】

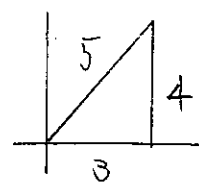
③ α の動径が第2象限、 β の動径が第1象限にあり、 $\sin \alpha = \frac{2}{3}$, $\cos \beta = \frac{3}{5}$ のとき、

$\sin(\alpha - \beta)$ と $\cos(\alpha + \beta)$ を求めよ。



$x^2 + 4 = 9$
 $x = \sqrt{5}$

$\cos \alpha = -\frac{\sqrt{5}}{3}$



$x^2 + 9 = 25$
 $x = 4$

$\sin \beta = \frac{4}{5}$

$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

$= \frac{2}{3} \cdot \frac{3}{5} + \frac{\sqrt{5}}{3} \cdot \frac{4}{5}$

$= \frac{6 + 4\sqrt{5}}{15}$

$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

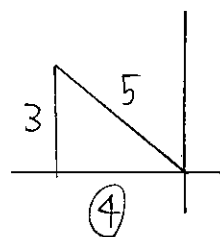
$= -\frac{\sqrt{5}}{3} \cdot \frac{3}{5} - \frac{2}{3} \cdot \frac{4}{5}$

$= \frac{-3\sqrt{5} - 8}{15}$

【2倍角の公式・半角の公式】

④ $0 < \alpha < \frac{\pi}{2}$ で $\sin \alpha = \frac{3}{5}$ のとき、次の値を求めよ。

(1) $\cos \alpha$



$\cos \alpha = -\frac{4}{5}$

(2) $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

$= 2 \cdot \frac{3}{5} \cdot \left(-\frac{4}{5}\right)$

$= -\frac{24}{25}$

(3) $\cos 2\alpha = 2 \cos^2 \alpha - 1$

$= 2 \cdot \frac{16}{25} - 1$

$= \frac{32}{25} - \frac{25}{25}$

$= \frac{7}{25}$

(4) $\sin \frac{\alpha}{2}$

$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$

$= \frac{1 + \frac{4}{5}}{2}$

$= \frac{\frac{9}{5}}{2} = \frac{9}{10}$

$\sin \frac{\alpha}{2} > 0 \neq y$

$\sin \frac{\alpha}{2} = \frac{3}{\sqrt{10}}$

$= \frac{3\sqrt{10}}{10}$

三角関数④

【2倍角の公式・半角の公式[2]】

5 次の値を求めよ。

(1) $\tan \alpha = 3$ のとき, $\tan 2\alpha$ の値

$$\begin{aligned} \tan(\alpha + \alpha) &= \frac{\tan \alpha + \tan \alpha}{1 - \tan \alpha \tan \alpha} \\ &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{6}{1 - 9} = -\frac{3}{4} \end{aligned}$$

(2) $\frac{\pi}{2} < \alpha < \pi$ で $\cos \alpha = -\frac{2}{3}$ のとき, $\tan \frac{\alpha}{2}$ の値

$$\begin{aligned} \tan^2 \frac{\alpha}{2} &= \frac{1 - \cos \alpha}{1 + \cos \alpha} \\ &= \frac{1 + \frac{2}{3}}{1 - \frac{2}{3}} = \frac{\frac{5}{3}}{\frac{1}{3}} = 5 \end{aligned}$$

【半角の公式】

6 半角の公式を使って, 次の値を求めよ。

(1) $\sin \frac{\pi}{8}$

$$\begin{aligned} \sin^2 \frac{\pi}{8} &= \frac{1 - \cos \frac{\pi}{4}}{2} \\ &= \frac{1 - \frac{1}{\sqrt{2}}}{2} \\ &= \frac{\sqrt{2} - 1}{2\sqrt{2}} = \frac{2 - \sqrt{2}}{4} \end{aligned}$$

$\sin \frac{\pi}{8} > 0$ より
 $\sin \frac{\pi}{8} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

(2) $\cos \frac{3\pi}{8}$

$$\begin{aligned} \cos^2 \frac{3\pi}{8} &= \frac{1 + \cos \frac{3\pi}{4}}{2} \\ &= \frac{1 - \frac{1}{\sqrt{2}}}{2} \\ &= \frac{\sqrt{2} - 1}{2\sqrt{2}} = \frac{2 - \sqrt{2}}{4} \end{aligned}$$

$\cos \frac{3\pi}{8} > 0$ より
 $\cos \frac{3\pi}{8} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

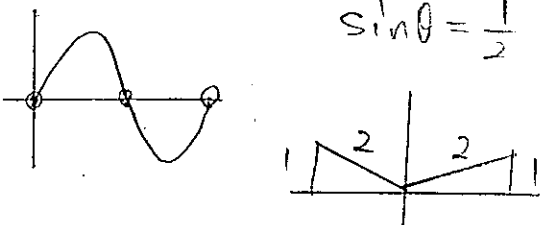
【2倍角の公式と方程式】

7 $0 \leq \theta < 2\pi$ のとき, 次の方程式を解け。

(1) $\cos 2\theta + \sin \theta = 1$

$$\begin{aligned} 1 - 2\sin^2 \theta + \sin \theta &= 1 \\ 2\sin^2 \theta - \sin \theta &= 0 \\ \sin \theta (2\sin \theta - 1) &= 0 \end{aligned}$$

$$\begin{aligned} \sin \theta = 0 & \quad 2\sin \theta - 1 \\ \sin \theta = \frac{1}{2} \end{aligned}$$

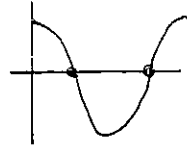


$\theta = 0, \pi$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\theta = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

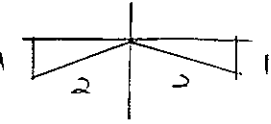
(2) $\sin 2\theta + \cos \theta = 0$

$$\begin{aligned} 2\sin \theta \cos \theta + \cos \theta &= 0 \\ \cos \theta (2\sin \theta + 1) &= 0 \end{aligned}$$

$$\cos \theta = 0 \quad 2\sin \theta + 1 = 0$$



$$\sin \theta = -\frac{1}{2}$$



$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

よって

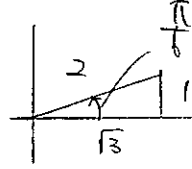
$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

【三角関数の合成】

8 次の式を $r \sin(\theta + \alpha)$ の形に表せ。ただし, $-\pi < \alpha < \pi$ とする。

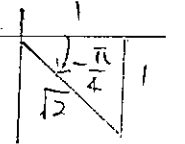
(1) $\sqrt{3} \sin \theta + \cos \theta$

$$= 2 \sin\left(\theta + \frac{\pi}{6}\right)$$



(2) $\sin \theta - \cos \theta$

$$= \sqrt{2} \sin\left(\theta - \frac{\pi}{4}\right)$$



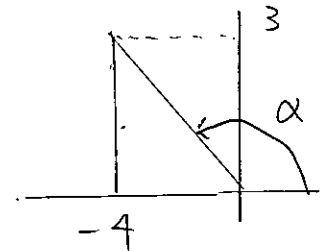
【三角関数の合成と最大・最小】

9 次の関数の最大値, 最小値を求めよ。

$$y = \sqrt{3} \sin x + \cos x$$

$$= 5 \sin(x + \alpha)$$

$$\begin{aligned} \text{Max } &5 \\ \text{min } &-5 \end{aligned}$$



【三角関数の合成と方程式】

10 $0 \leq x < 2\pi$ のとき, 次の方程式を解け。

$$\sin x + \sqrt{3} \cos x = 1$$

$$2 \sin\left(x + \frac{\pi}{3}\right) = 1$$

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$$

$$x + \frac{\pi}{3} = t \quad t \in \mathbb{R}$$

$$\sin t = \frac{1}{2}$$

$$t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{11\pi}{6}$$

30° 150°
390°

$$\begin{aligned} 0 \leq x < 2\pi \\ \downarrow \\ 0 + \frac{\pi}{3} \leq x + \frac{\pi}{3} < 2\pi + \frac{\pi}{3} \\ \downarrow \\ \frac{\pi}{3} \leq t < \frac{7\pi}{3} \\ 60^\circ \sim 420^\circ \end{aligned}$$

$$t = \frac{13\pi}{6} \text{ のとき}$$

$$t = \frac{5\pi}{6} \text{ のとき}$$

$$x + \frac{\pi}{3} = \frac{13\pi}{6}$$

$$x + \frac{\pi}{3} = \frac{5\pi}{6}$$

$$x = \frac{11\pi}{6}$$

$$x = \frac{1}{2}\pi$$

$$x = \frac{1}{2}\pi, \frac{11\pi}{6}$$