

漸化式 (公式)

等差型

$$a_{n+1} - a_n = d \quad \Rightarrow \quad \text{公差 } d \text{ の等差数列}$$

$$a_1 = -3, a_{n+1} = a_n + 4$$

解き方

初項 -3 , 公差 4 の等比数列
 $a_n = -3 + (n-1) \cdot 4$

等比型

$$a_{n+1} = r a_n \quad \Rightarrow \quad \text{公比 } r \text{ の等比数列}$$

$$a_1 = 1, a_{n+1} = 2a_n$$

解き方

初項 1 , 公比 2 の等比数列
 $a_n = 2^{n-1}$

階差型

$$a_{n+1} = a_n + f(n) \quad \Rightarrow \quad f(n) \text{ が一般項の階差数列}$$

$$a_1 = 1, a_{n+1} = a_n + 3^n$$

解き方

$n \geq 2$ のとき k に変える
 $a_n = a_1 + \sum_{k=1}^{n-1} 3^k$

特性方程式型

$$a_{n+1} = p a_n + q \quad \Rightarrow \quad \text{特性方程式}$$

$$a_1 = 6, a_{n+1} = 4a_n - 3$$

解き方

$$\begin{array}{r} a_{n+1} = 4a_n - 3 \\ -) \quad 1 = 4 \cdot 1 - 3 \\ \hline a_{n+1} - 1 = 4(a_n - 1) \end{array} \quad \begin{array}{l} \alpha = 4\alpha - 3 \\ \alpha = 1 \end{array}$$

特性階差型

$$a_{n+1} = p a_n + f(n) \quad \Rightarrow \quad a_{n+2} = p a_{n+1} + f(n+1) \text{ と引き算}$$

$$a_1 = 1, a_{n+1} = 3a_n + 4n \quad \dots\dots ①$$

解き方

$$\begin{array}{l} a_{n+2} = 3a_{n+1} + 4(n+1) \quad \dots\dots ② \\ ② - ① \\ \hline a_{n+2} - a_{n+1} = 3(a_{n+1} - a_n) + 4 \end{array}$$

$b_n = a_{n+1} - a_n$ とおいて特性方程式へ

分数型

$$a_{n+1} = \frac{a_n}{p a_n + q} \quad \Rightarrow \quad \text{逆数をとる}$$

$$a_1 = \frac{1}{5}, a_{n+1} = \frac{a_n}{4a_n - 1}$$

解き方

$$\begin{array}{l} \frac{1}{a_{n+1}} = \frac{4a_n - 1}{a_n} \\ \frac{1}{a_{n+1}} = -\frac{1}{a_n} + 4 \end{array} \quad b_n = \frac{1}{a_n} \text{ とおいて特性方程式へ}$$

指数型

$$a_{n+1} = p a_n + q^n \quad \Rightarrow \quad q^{n+1} \text{ で割る}$$

$$a_1 = 3, a_{n+1} = 2a_n + 3^{n+1}$$

解き方

$$\begin{array}{l} \frac{a_{n+1}}{3^{n+1}} = \frac{2a_n}{3^{n+1}} + 1 \\ \frac{a_{n+1}}{3^{n+1}} = \frac{2}{3} \cdot \frac{a_n}{3^n} + 1 \end{array} \quad b_n = \frac{a_n}{3^n} \text{ とおいて特性方程式へ}$$

対数型

$$a_{n+1} = p a_n^q \quad \Rightarrow \quad \text{底が } p \text{ の対数をとる}$$

$$a_1 = 1, a_{n+1} = 2\sqrt{a_n}$$

解き方

$$\begin{array}{l} \log_2 a_{n+1} = \log_2 2\sqrt{a_n} \\ \log_2 a_{n+1} = \log_2 \sqrt{a_n} + 1 \\ \log_2 a_{n+1} = \frac{1}{2} \log_2 a_n + 1 \end{array} \quad b_n = \log_2 a_n \text{ とおいて特性方程式へ}$$

階比型

$$a_n = f(n) a_{n-1} \quad \Rightarrow \quad a_n = f(n) f(n-1) \dots f(2) a_1$$

$$a_1 = \frac{1}{2}, (n+1)a_n = (n-1)a_{n-1} \quad (n \geq 2)$$

解き方

$$\begin{array}{l} a_n = \frac{n-1}{n+1} a_{n-1} \\ a_n = \frac{n-1}{n+1} \cdot \frac{n-2}{n} \cdot \frac{n-3}{n-1} \cdot \frac{n-4}{n-2} \dots \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} a_1 \end{array}$$

部分分数分解型

$$a_{n+1} = f(n) a_n + q \quad \Rightarrow \quad n \text{ の式で両辺割る}$$

$$a_1 = 2, n a_{n+1} = (n+1) a_n + 1$$

解き方

両辺を $n(n+1)$ で割ると

$$\frac{a_n}{n+1} = \frac{a_n}{n} + \frac{1}{n(n+1)} \quad b_n = \frac{a_n}{n} \text{ とおいて階差数列へ}$$