**【弧度法】**

$\boxed{1}$ 次の角を，度数は弧度に，弧度は度数に，それぞれ書き直せ。

(1) 30°　(2) 225°　(3) 123°　(4) $\frac{4}{3}\pi$　(5) $\frac{\pi}{60}$

$\frac{1}{6}\pi$　$\frac{5}{4}\pi$　$\frac{41}{60}\pi$　$240°$　$3°$

**【扇形の長さと面積】**

$\boxed{2}$ 次のような扇形の弧の長さと面積を求めよ。

(1) 半径 4，中心角 $\frac{\pi}{3}$　$\frac{60}{360}=\frac{1}{6}$

円

周 $2\pi \cdot 4 = 8\pi$

面積 $\pi \cdot 4^2 = 16\pi$

扇形

弧 $8\pi \times \frac{1}{6} = \frac{4}{3}\pi$

面積 $16\pi \times \frac{1}{6} = \frac{8}{3}\pi$

(2) 半径 6，中心角 $\frac{7}{6}\pi$　$\frac{210}{360}=\frac{7}{12}$

円

周 $2\pi \cdot 6 = 12\pi$

面積 $\pi \cdot 6^2 = 36\pi$

扇形

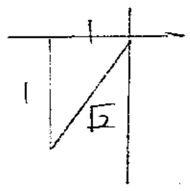
弧 $12\pi \times \frac{7}{12} = 7\pi$

面積 $36\pi \times \frac{7}{12} = 21\pi$

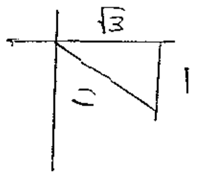
**【三角関数の値】**

$\boxed{3}$ 次の値を，それぞれ求めよ。

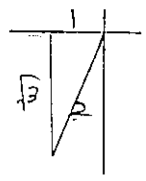
(1) $\cos\frac{5}{4}\pi = -\frac{1}{\sqrt{2}}$



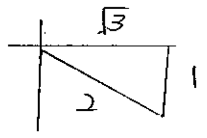

(2) $\sin\frac{11}{6}\pi = -\frac{1}{2}$




(3) $\tan\frac{4}{3}\pi = \sqrt{3}$



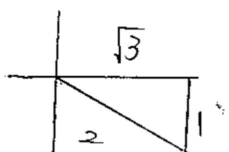

(4) $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$




(5) $\cos\left(-\frac{13}{6}\pi\right)$

$=\cos\left(-\frac{\pi}{6}\right)$

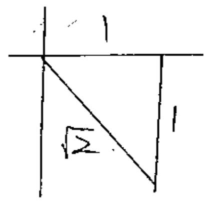
$=\frac{\sqrt{3}}{2}$




(6) $\tan\left(-\frac{9}{4}\pi\right)$

$=\tan\left(-\frac{\pi}{4}\right)$

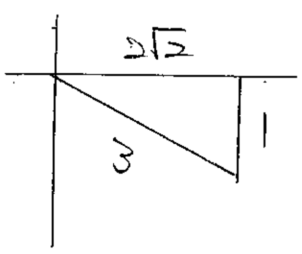
$=-1$




**【三角関数の相互関係】**

$\boxed{4}$ 次の値を求めよ。

(1) $\theta$ の動径が第 4 象限にあり，$\sin\theta=-\frac{1}{3}$ のとき，$\cos\theta$ と $\tan\theta$ の値　+　−




$x^2+1=9$

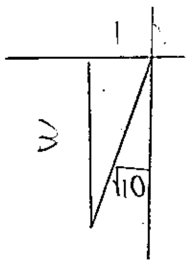
$x^2=8$

$x=2\sqrt{2}$

$\cos\theta = \frac{2\sqrt{2}}{3}$

$\tan\theta = -\frac{1}{2\sqrt{2}}$

(2) $\theta$ の動径が第 3 象限にあり，$\tan\theta=\frac{3}{1}$ のとき，$\sin\theta$ と $\cos\theta$ の値　−　−




$x^2=1+9$

$=10$

$x=\sqrt{10}$

$\sin\theta = -\frac{3}{\sqrt{10}}$

$\cos\theta = -\frac{1}{\sqrt{10}}$

**【相互関係による式の値】**

$\boxed{5}$ $\sin\theta+\cos\theta=\frac{1}{2}$ のとき，次の式の値を求めよ。

(1) $\sin\theta\cos\theta$

$(\sin\theta+\cos\theta)^2=\frac{1}{4}$

$\underbrace{\sin^2+2\sin\theta\cos\theta+\cos^2\theta}$ $=\frac{1}{4}$

1

$2\sin\theta\cos\theta = \frac{1}{4}-1$

$\sin\theta\cos\theta = -\frac{3}{8}$

(2) $\sin^3\theta+\cos^3\theta$

$=(\sin\theta+\cos\theta)^3-3\sin\theta\cos\theta(\sin\theta+\cos\theta)$

$=\left(\frac{1}{2}\right)^3-3\cdot\left(-\frac{3}{8}\right)\cdot\frac{1}{2}$

$=\frac{1}{8}+\frac{9}{16} = \frac{11}{16}$

(3) $\sin\theta-\cos\theta$

$(\sin\theta-\cos\theta)^2 = \underbrace{\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta}$

1

$=1-2\cdot\left(-\frac{3}{8}\right)$

$=\frac{7}{4}$

よって

$\sin\theta-\cos\theta = \pm\frac{\sqrt{7}}{2}$

**【相互関係による等式の証明】**

$\boxed{6}$ 等式 $\tan^2\theta-\sin^2\theta=\tan^2\theta\sin^2\theta$ を証明せよ。

$(左辺) = \frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta$

$=\sin^2\theta\left(\frac{1}{\cos^2\theta}-1\right)$

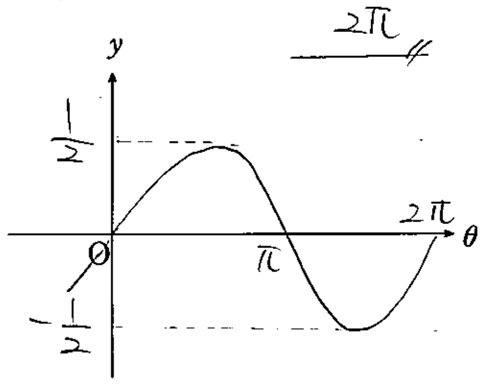
$=\sin^2\theta\tan^2\theta = (右辺)$

よって　(左辺) = (右辺)

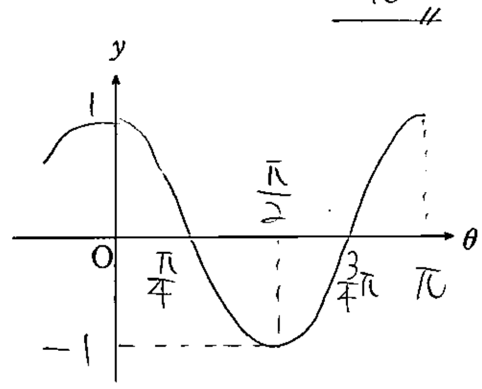
**【三角関数のグラフ[1]】**

$\boxed{7}$ 次の関数のグラフをかけ。また，その周期を求めよ。

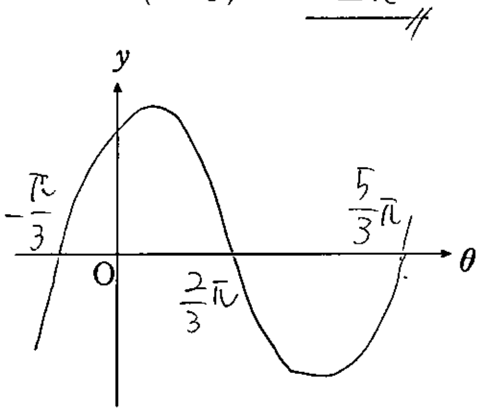
(1) $y=\frac{1}{2}\sin\theta$　$2\pi$



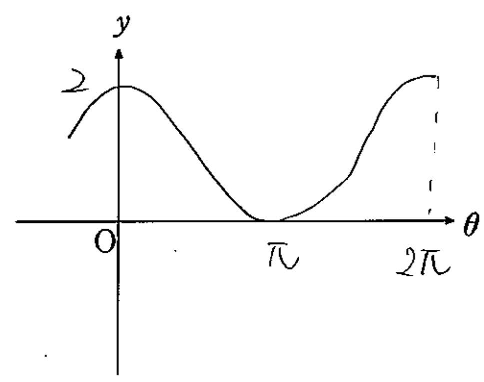

(2) $y=\cos 2\theta$　$\pi$




(3) $y=\sin\left(\theta+\frac{\pi}{3}\right)$　$2\pi$



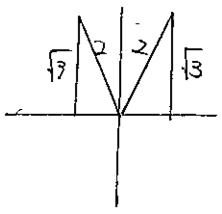

(4) $y=\cos\theta+1$　$2\pi$



## 【三角関数を含む方程式】

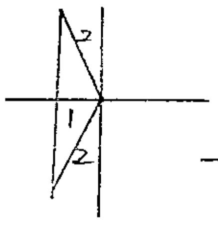
$\boxed{8}$ $0 \leqq \theta < 2\pi$ のとき，次の方程式を解け。

(1) $\sin\theta = \frac{\sqrt{3}}{2}$



$\theta = \frac{\pi}{3}, \frac{2}{3}\pi$

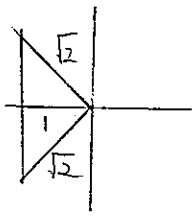
(2) $2\cos\theta + 1 = 0$



$\theta = \frac{2}{3}\pi, \frac{4}{3}\pi$

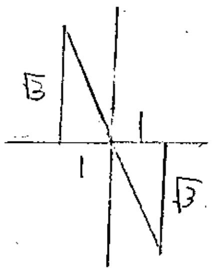
(3) $\cos\theta = -\frac{\sqrt{2}}{2}$

$\cos\theta = -\frac{1}{\sqrt{2}}$



$\theta = \frac{3}{4}\pi, \frac{5}{4}\pi$

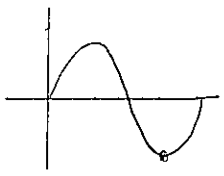
(4) $\tan\theta = -\frac{\sqrt{3}}{1}$



$\theta = \frac{2}{3}\pi, \frac{5}{3}\pi$

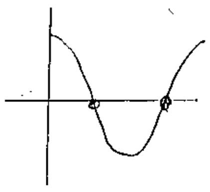
(5) $\sin\theta + 1 = 0$

$\sin\theta = -1$



$\theta = \frac{3}{2}\pi$

(6) $\cos\theta = 0$

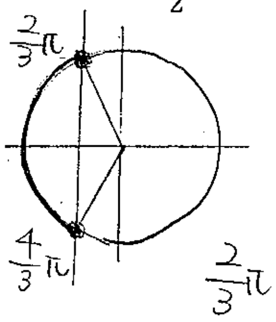


$\theta = \frac{\pi}{2}, \frac{3}{2}\pi$

## 【三角関数を含む不等式】

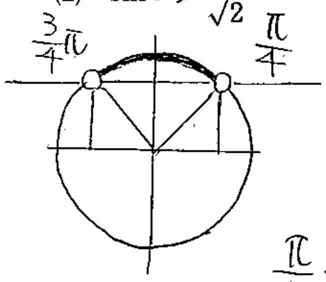
$\boxed{9}$ $0 \leqq \theta < 2\pi$ のとき，次の不等式を解け。

(1) $\cos\theta \leqq -\frac{1}{2}$



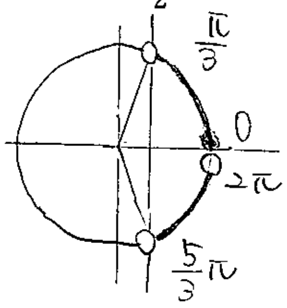
$\frac{2}{3}\pi \leqq \theta \leqq \frac{4}{3}\pi$

(2) $\sin\theta > \frac{1}{\sqrt{2}}$



$\frac{\pi}{4} < \theta < \frac{3}{4}\pi$

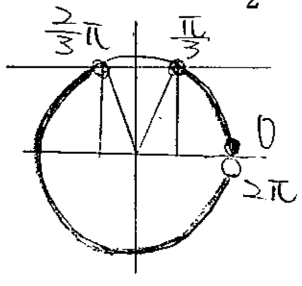
(3) $\cos\theta > \frac{1}{2}$



$0 \leqq \theta < \frac{\pi}{3}$

$\frac{5}{3}\pi < \theta < 2\pi$

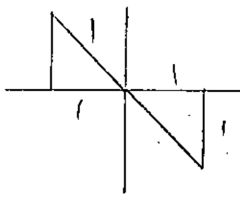
(4) $\sin\theta \leqq \frac{\sqrt{3}}{2}$



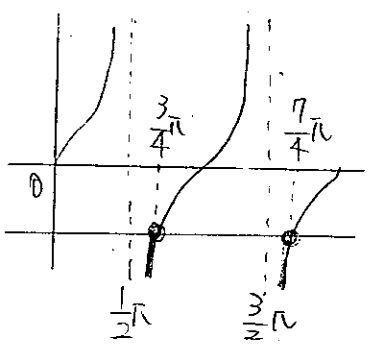
$0 \leqq \theta \leqq \frac{\pi}{3}$

$\frac{2}{3}\pi \leqq \theta < 2\pi$

(5) $\tan\theta \leqq -\frac{1}{1}$



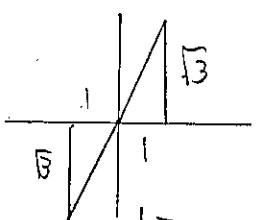
$\theta = \frac{3}{4}\pi, \frac{7}{4}\pi$



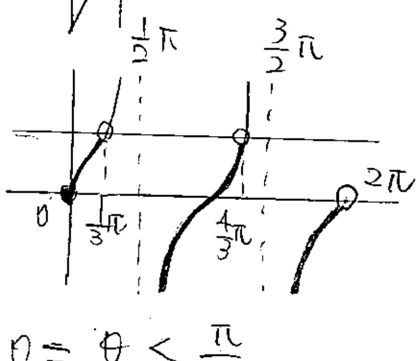
$\frac{1}{2}\pi < \theta \leqq \frac{3}{4}\pi$

$\frac{3}{2}\pi < \theta \leqq \frac{7}{4}\pi$

(6) $\tan\theta < \frac{\sqrt{3}}{1}$



$\theta = \frac{1}{3}\pi, \frac{4}{3}\pi$



$0 \leqq \theta < \frac{\pi}{3}$

$\frac{\pi}{2} < \theta < \frac{4}{3}\pi$

$\frac{3}{2}\pi < \theta < 2\pi$

## 【三角関数を含む方程式[1]】

$\boxed{10}$ $0 \leqq \theta < 2\pi$ のとき，方程式 $\sin\left(\theta + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$ を解け。

$\theta + \frac{\pi}{3} = t$ とおく

$\sin t = \frac{1}{\sqrt{2}}$

$0 \leqq \theta < 2\pi$

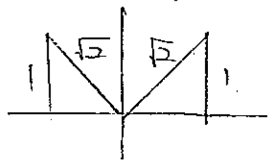
↓

$0 + \frac{\pi}{3} \leqq \theta + \frac{\pi}{3} < 2\pi + \frac{\pi}{3}$

↓

$\frac{\pi}{3} \leqq t < \frac{7}{3}\pi$

60° ～ 420°



$t = \cancel{\frac{\pi}{4}}, \frac{3}{4}\pi$

~~45°~~ 135°

$\frac{9}{4}\pi$

$t = \frac{3}{4}\pi$ のとき，$t = \frac{9}{4}\pi$ のとき

$\theta + \frac{\pi}{3} = \frac{3}{4}\pi$　　$\theta + \frac{\pi}{3} = \frac{9}{4}\pi$

$\theta = \frac{5}{12}\pi$　　$\theta = \frac{23}{12}\pi$

## 【三角関数を含む方程式[2]】

$\boxed{11}$ $0 \leqq \theta < 2\pi$ のとき，方程式 $2\cos^2\theta + 3\cos\theta - 2 = 0$ を解け。

$\cos\theta = t$ とおく

$2t^2 + 3t - 2 = 0$

$(2t-1)(t+2) = 0$

$t = \cancel{-2}, \frac{1}{2}$

不適

$0 \leqq \theta < 2\pi$

↓

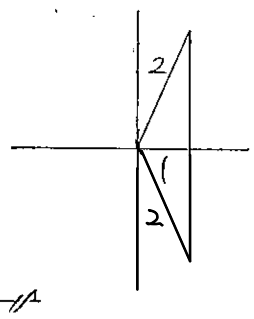
$-1 \leqq \cos\theta \leqq 1$

↓

$-1 \leqq t \leqq 1$

$t = \frac{1}{2}$ のとき

$\cos\theta = \frac{1}{2}$



$\theta = \frac{\pi}{3}, \frac{5}{3}\pi$

## 【三角関数を含む関数の最大・最小】

$\boxed{12}$ $0 \leqq \theta < 2\pi$ のとき，関数 $y = -\sin^2\theta - \cos\theta + 1$ の最大値と最小値を求めよ。また、そのときの $\theta$ の値を求めよ。

$y = -(1-\cos^2\theta) - \cos\theta + 1$

$y = \cos^2\theta - \cos\theta$

$\cos\theta = t$ とおく

$y = t^2 - t$

$= \left(t - \frac{1}{2}\right)^2 - \frac{1}{4}$

$0 \leqq \theta < 2\pi$

↓

$-1 \leqq \cos\theta < 1$

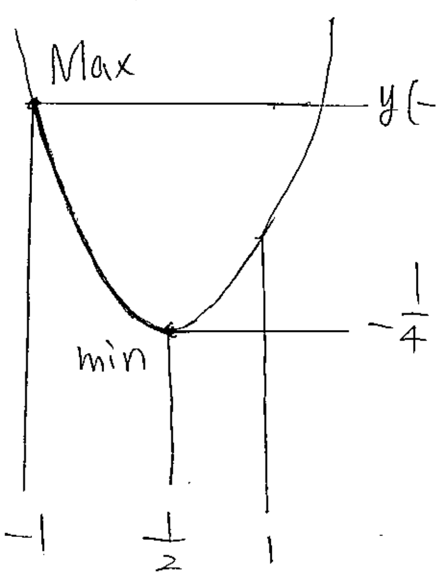
↓

$-1 \leqq t < 1$

よって．

Max 2　$(t = -1)$

min $-\frac{1}{4}$　$(t = \frac{1}{2})$




$\cos\theta = -1$

$\theta = \pi$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5}{3}\pi$

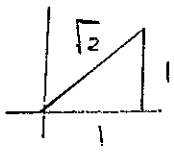
従って

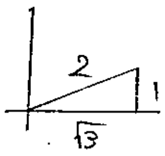
Max 2　$(\theta = \pi)$

min $-\frac{1}{4}$　$(\theta = \frac{\pi}{3}, \frac{5}{3}\pi)$

**【加法定理】**

$\boxed{13}$ 加法定理を用いて，次の三角関数の値を求めよ。




(1) $\sin 75°$ $= \sin(45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$ //

(2) $\cos 15°$ $= \cos(45° - 30°)$

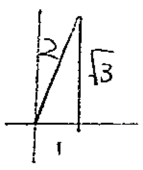
$= \cos 45° \cos 30° + \sin 45° \sin 30°$

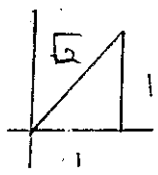
$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$ //

(3) $\tan 105°$ $= \tan(60° + 45°)$




$= \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$

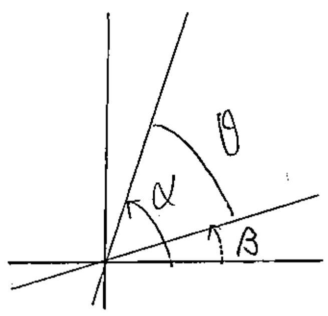
$= \frac{1+\sqrt{3}}{1-\sqrt{3}}$

$= \frac{(1+\sqrt{3})^2}{(1-\sqrt{3})(1+\sqrt{3})}$ $= \frac{4+2\sqrt{3}}{-2}$

$= \frac{4+2\sqrt{3}}{1-3}$ $= -2-\sqrt{3}$ //

**【２直線のなす角】**

$\boxed{14}$ ２直線 $y=2x-1$，$y=\frac{1}{3}x+1$ のなす角 $\theta$ を求めよ。ただし，$0<\theta<\frac{\pi}{2}$ とする。




$\tan\alpha = 2$

$\tan\beta = \frac{1}{3}$

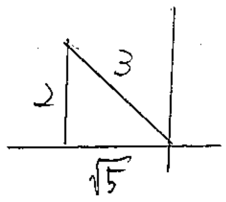
$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$ $= \frac{\frac{5}{3}}{\frac{5}{3}}$

$= \frac{2-\frac{1}{3}}{1+\frac{2}{3}}$ $= 1$

よって $\theta = \frac{\pi}{4}$ //

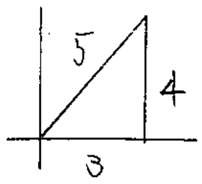
**【加法定理の応用】**

$\boxed{15}$ $\alpha$ の動径が第２象限，$\beta$ の動径が第１象限にあり，$\sin\alpha=\frac{2}{3}$，$\cos\beta=\frac{3}{5}$ のとき，$\sin(\alpha-\beta)$ と $\cos(\alpha+\beta)$ を求めよ。




$x^2+4=9$

$x=\sqrt{5}$




$x^2+9=25$

$x=4$

$\cos\alpha = -\frac{\sqrt{5}}{3}$ $\sin\beta = \frac{4}{5}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{2}{3} \cdot \frac{3}{5} + \frac{\sqrt{5}}{3} \cdot \frac{4}{5}$

$= \frac{6+4\sqrt{5}}{15}$ //

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

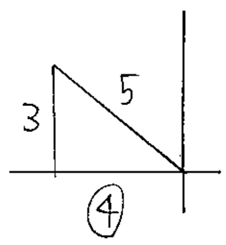
$= -\frac{\sqrt{5}}{3} \cdot \frac{3}{5} - \frac{2}{3} \cdot \frac{4}{5}$

$= \frac{-3\sqrt{5}-8}{15}$ //

**【２倍角の公式･半角の公式】**

$\boxed{16}$ $\frac{\pi}{2}<\alpha<\pi$ で $\sin\alpha=\frac{3}{5}$ のとき，次の値を求めよ。

(1) $\cos\alpha$




$\cos\alpha = -\frac{4}{5}$ //

(2) $\sin 2\alpha$ $= 2\sin\alpha\cos\alpha$

$= 2 \cdot \frac{3}{5} \cdot \left(-\frac{4}{5}\right)$

$= -\frac{24}{25}$ //

(3) $\cos 2\alpha$ $= 2\cos^2\alpha - 1$

$= 2 \cdot \frac{16}{25} - 1$

$= \frac{32}{25} - \frac{25}{25}$

$= \frac{7}{25}$ //

(4) $\sin\frac{\alpha}{2}$

$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$

$= \frac{1+\frac{4}{5}}{2}$

$= \frac{\frac{9}{5}}{2} = \frac{9}{10}$

$\sin\frac{\alpha}{2} > 0$ より

$\sin\frac{\alpha}{2} = \frac{3}{\sqrt{10}}$

$= \frac{3\sqrt{10}}{10}$ //

# 三角関数④

**【２倍角の公式･半角の公式[2]】**

$\boxed{17}$ 次の値を求めよ。

(1) $\tan\alpha=3$ のとき，$\tan 2\alpha$ の値

$$\tan(\alpha+\alpha)=\frac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}$$

$$=\frac{2\tan\alpha}{1-\tan^2\alpha}=\frac{6}{1-9}=-\frac{3}{4}$$

(2) $\frac{\pi}{2}<\alpha<\pi$ で $\cos\alpha=-\frac{2}{3}$ のとき，$\tan\frac{\alpha}{2}$ の値

$$\tan^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{1+\cos\alpha}$$

(+)

$\tan\frac{\alpha}{2}>0$ より

$\tan\frac{\alpha}{2}=\sqrt{5}$

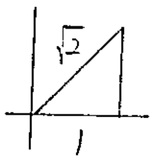
$$=\frac{1+\frac{2}{3}}{1-\frac{2}{3}}=\frac{\frac{5}{3}}{\frac{1}{3}}=5$$

**【半角の公式】**

$\boxed{18}$ 半角の公式を使って，次の値を求めよ。

(1) $\sin\frac{\pi}{8}$

$$\sin^2\frac{\pi}{8}=\frac{1-\cos\frac{\pi}{4}}{2}$$




$$=\frac{1-\frac{1}{\sqrt{2}}}{2}$$

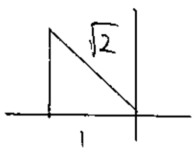
$\sin\frac{\pi}{8}>0$ より

$$=\frac{\sqrt{2}-1}{2\sqrt{2}}=\frac{2-\sqrt{2}}{4}$$

$\sin\frac{\pi}{8}=\frac{\sqrt{2-\sqrt{2}}}{2}$

(2) $\cos\frac{3}{8}\pi$

$$\cos^2\frac{3}{8}\pi=\frac{1+\cos\frac{3}{4}\pi}{2}$$




$$=\frac{1-\frac{1}{\sqrt{2}}}{2}$$

$\cos\frac{3}{8}\pi>0$ より

$$=\frac{\sqrt{2}-1}{2\sqrt{2}}=\frac{2-\sqrt{2}}{4}$$

$\cos\frac{3}{8}\pi=\frac{\sqrt{2-\sqrt{2}}}{2}$

**【２倍角の公式と方程式】**

$\boxed{19}$ $0\leqq\theta<2\pi$ のとき，次の方程式を解け。

(1) $\cos 2\theta+\sin\theta=1$

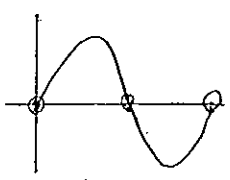
$1-2\sin^2\theta+\sin\theta=1$

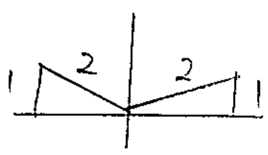
$2\sin^2\theta-\sin\theta=0$

$\sin\theta(2\sin\theta-1)=0$

$\sin\theta=0$　　$2\sin\theta-1$

$\sin\theta=\frac{1}{2}$






$\theta=0,\pi$　　$\theta=\frac{\pi}{6},\frac{5}{6}\pi$

よって

$\theta=0,\pi,\frac{\pi}{6},\frac{5}{6}\pi$

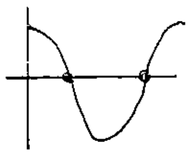
(2) $\sin 2\theta+\cos\theta=0$

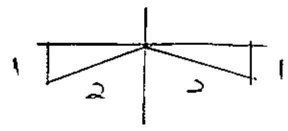
$2\sin\theta\cos\theta+\cos\theta=0$

$\cos\theta(2\sin\theta+1)=0$

$\cos\theta=0$　　$2\sin\theta+1=0$

$\sin\theta=-\frac{1}{2}$






$\theta=\frac{1}{2}\pi,\frac{3}{2}\pi$

$\theta=\frac{7}{6}\pi,\frac{11}{6}\pi$

よって

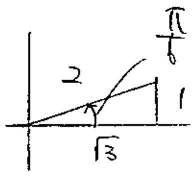
$\theta=\frac{1}{2}\pi,\frac{3}{2}\pi,\frac{7}{6}\pi,\frac{11}{6}\pi$

**【三角関数の合成】**

$\boxed{20}$ 次の式を $r\sin(\theta+\alpha)$ の形に表せ。ただし，$-\pi<\alpha<\pi$ とする。

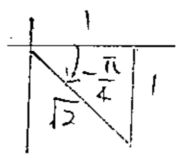
(1) $\sqrt{3}\sin\theta+\cos\theta$

$=2\sin\left(\theta+\frac{\pi}{6}\right)$




(2) $\sin\theta-\cos\theta$

$=\sqrt{2}\sin\left(\theta-\frac{\pi}{4}\right)$

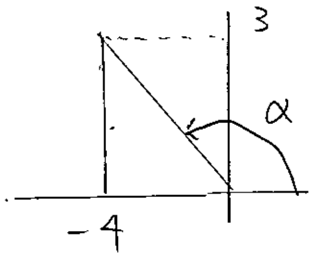



**【三角関数の合成と最大･最小】**

$\boxed{21}$ 次の関数の最大値，最小値を求めよ。

$y=-4\sin x+3\cos x$

$=5\sin(x+\alpha)$




Max 5

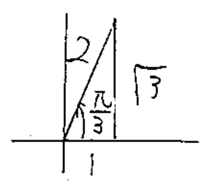
min -5

**【三角関数の合成と方程式】**

$\boxed{22}$ $0\leqq x<2\pi$ のとき，次の方程式を解け。

$\sin x+\sqrt{3}\cos x=1$

$2\sin\left(x+\frac{\pi}{3}\right)=1$




$\sin\left(x+\frac{\pi}{3}\right)=\frac{1}{2}$

$x+\frac{\pi}{3}=t$ とおく

$\sin t=\frac{1}{2}$

$t=\cancel{\frac{\pi}{6}},\frac{5}{6}\pi$

$\cancel{30^\circ}$　$150^\circ$

$\frac{13}{6}\pi$

$390^\circ$

$$\left(\begin{array}{c}0\leqq x<2\pi\\ \downarrow\\ 0+\frac{\pi}{3}\leqq x+\frac{\pi}{3}<2\pi+\frac{\pi}{3}\\ \downarrow\\ \frac{\pi}{3}\leqq t<\frac{7}{3}\pi\\ 60^\circ\sim420^\circ\end{array}\right)$$

$t=\frac{13}{6}\pi$ のとき　　$t=\frac{5}{6}\pi$ のとき

$x+\frac{\pi}{3}=\frac{13}{6}\pi$　　$x+\frac{\pi}{3}=\frac{5}{6}\pi$

$x=\frac{11}{6}\pi$　　$x=\frac{1}{2}\pi$

よって

$x=\frac{1}{2}\pi,\frac{11}{6}\pi$